$$\left\{ \frac{\sin n}{e^n} \right\}$$

$-1 \le \sin n \le 1$ FOR $n \in \mathbb{Z}^+$

(1) $-\frac{1}{e^n} \le \frac{\sin n}{e^n} \le \frac{1}{e^n}$

(1) $\lim_{n \to \infty} -\frac{1}{e^n} = \lim_{n \to \infty} \frac{1}{e^n} = 0$

BY SQUEEZE THEOREM

(1) $\lim_{n \to \infty} \frac{\sin n}{e^n} = 0$

$$\sum_{n=1}^{\infty} 3^{\frac{1-n}{n}}$$

$$\lim_{n\to\infty} \frac{1-n}{n} = -1 \quad 1 \quad 1$$

$$\text{so } \lim_{n\to\infty} 3^{\frac{1-n}{n}} = 3^{-1} = \frac{1}{3} \neq 0$$

BY DIVERGENCE TEST

$$\sum_{n=1}^{\infty} 3^{\frac{1-n}{n}} \text{ DIVERGES}$$

1

$$\sum_{n=1}^{\infty} \frac{2+2^{2n}}{5^n}$$

$$= \sum_{n=1}^{\infty} 2\cdot\left(\frac{1}{5}\right)^n + \sum_{n=1}^{\infty}\left(\frac{4}{5}\right)^n$$

(BOTH GEOMETRIC SERIES
$r = \frac{1}{5}$ AND $r = \frac{4}{5}$)

$$= \underbrace{\frac{\frac{2}{5}}{1-\frac{1}{5}}}_{} \; 1 + \underbrace{\frac{\frac{4}{5}}{1-\frac{4}{5}}}_{} \; 1$$

$$= \frac{1}{2} + 4$$

$$= \underbrace{\frac{9}{2}}_{} \; 1$$

$$\left\{ \frac{n}{\sqrt{1+4n^2}} \right\}$$

$$\lim_{n \to \infty} \frac{n}{\sqrt{1+4n^2}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$$

$$= \lim_{n \to \infty} \frac{1}{\sqrt{\frac{1}{n^2} + 4}} \quad 2$$

$$= \frac{1}{\sqrt{0+4}}$$

$$= \frac{1}{2} \quad 1$$

Consider the following statements. SCORE: _____ / 3 PTS

(i) If $\{a_n\}$ has limit 0, then $\sum_{n=1}^{\infty} a_n$ is convergent

(ii) If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\{a_n\}$ diverges

(iii) If $\{a_n\}$ is bounded, then $\{a_n\}$ converges

Which of the statements above are true ? **Circle the correct answer below.**

3

[a]	none are true	[b]	only (i) is true	[c]	only (ii) is true	[d]	only (iii) is true
[e]	only (i) and (ii) are true	[f]	only (i) and (iii) are true	[g]	only (ii) and (iii) are true	[h]	all are true

Find all values of x for which $\sum_{n=0}^{\infty} 2^{n+1}(3-x)^n$ is convergent. **<u>You do NOT need to find the sum.</u>** SCORE: _____ / 4 PTS

$= 2 + 2^2(3-x) + 2^3(3-x)^2 + \ldots$

GEOMETRIC SERIES

$r = 2(3-x)$

$|2(3-x)| < 1$ 2

$-1 < 2(3-x) < 1$

$-\frac{1}{2} < 3 - x < \frac{1}{2}$

$-\frac{7}{2} < -x < -\frac{5}{2}$

$\frac{5}{2} < x < \frac{7}{2}$ 2

Determine if each sequence below is increasing, decreasing or neither. SCORE: _____ / 5 PTS

Justify each answer using proper mathematical reasoning and/or algebra.

Your solutions must NOT use derivatives.

[a] $\left\{ \dfrac{3n-5}{4n-3} \right\}$

$$a_{n+1} - a_n = \frac{3(n+1)-5}{4(n+1)-3} - \frac{3n-5}{4n-3}$$

$$= \frac{3n-2}{4n+1} - \frac{3n-5}{4n-3} \quad 1$$

$$= \frac{11}{(4n+1)(4n-3)} \quad 1$$

$a_{n+1} - a_n > 0$ FOR $n \in \mathbb{Z}^+$

$a_{n+1} > a_n$ FOR $n \in \mathbb{Z}^+$ 1 EITHER STATEMENT IS OK

INCREASING 1

[b] $\left\{ \dfrac{5n-11}{2n-5} \right\} = \{2, 1, 4, \ldots\}$ $\frac{1}{2}$

NEITHER $\frac{1}{2}$